Durham University

CeREES
Centre for Research into Earth Energy Systems
energy today and tomorrow

Step-drawdown tests and the Forchheimer equation

Simon A. Mathias
Department of Earth Sciences, Durham University (Email: s.a.mathias@durham.ac.uk)

Is groundwater flow ever turbulent?

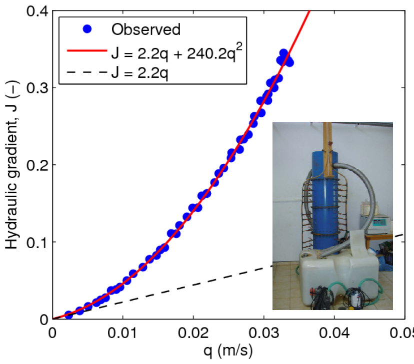


Figure 1: Results from a permeameter experiment on a quarry carbonate rock (after Moutsopoulos et al., 2009). Darcy's law (q linearly proportional to J) is known to work badly for high velocities.

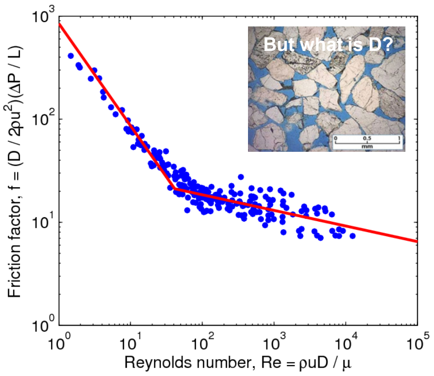
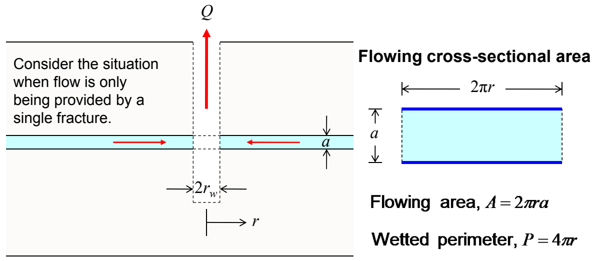


Figure 2: Friction factor against Reynolds number, Re, for a variety of porous media (after Chilton and Colburn, 1931). For pipe flow, turbulent flow occurs when Re > 2000. For porous media people say Re > 40.



Reynolds number, $\mathrm{Re} = \frac{4\rho u A}{\mu P} = \frac{4\rho u}{\mu}\left(\frac{2\pi r a}{4\pi r}\right) = \frac{2\rho u a}{\mu}$

But $u = \frac{Q}{A} = \frac{Q}{2\pi r a} \Rightarrow \mathrm{Re} = \frac{\rho Q}{\pi \mu r}$ So Re is independent of a?

Figure 3: A simple example showing that when one considers a scheme when water is produced from a single fracture, Re becomes independent of fracture aperture, a. The example illustrates one of many problems with applying Re to understanding groundwater flow.

Forchheimer equation

By consideration of data such as shown in Figure 1, Forchheimer (1901) suggested the quadratic alternative to Darcy's Law:

$$\frac{\mu q}{k} + \rho b q^2 + \frac{dP}{dx} = 0$$

where

k [L^2] is permeability
μ [ML^{-1}T^{-1}] is dynamic viscosity
P [ML^{-1}T^{-2}] is fluid pressure
x [L] is distance
ρ [ML^{-3}] is fluid density
b [L^{-1}] is the Forchheimer parameter

From a dimensional analysis Ward (1964) established that

$$b = f(k^{-1/2})$$

From an empirical analysis Geertsma (1974) proposed that

$$b = 0.005\phi^{-5.5}k^{-0.5}$$

where ϕ [-] is porosity.

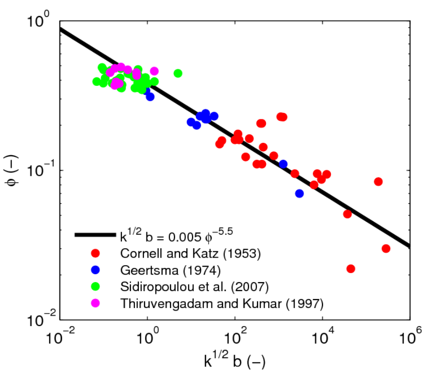


Figure 4: Permeameter data that supports the Geertsma correlation. Note that Geertsma only used his own data and that of Cornell and Katz for the linear regression.

Step drawdown tests

A common way to test a well is to pump at sequentially increasing rates; a step drawdown test. After a certain amount of time, drawdown in the well, s_w, reaches a quasi-steady value. These drawdowns are plotted against pumping rate, Q_w, and a quadratic is fitted (the Jacob Method):

$$s_w = AQ_w + BQ_w^2$$

where A and B are known as the formation loss and well loss factors, respectively. Comparison with the large-time solution for Forchheimer flow to a well (Mathias et al., 2008)

$$s_w \approx \frac{Q_w}{4\pi T}\left[\ln\left(\frac{4Tt}{Sr_w^2}\right) - 0.5772\right] + \frac{bQ_w^2}{(2\pi H)^2 r_w g}$$

where

H [L] is formation thickness
t [T] is time
$T = Hk\rho g/\mu$ [L^{2}T^{-1}] is transmissivity
$S = H\phi(c_r + c_w)\rho g$ [-] is storativity
r_w [L] is well radius
g [LT^{-2}] is gravity
c_r, c_w [M^{-1}LT^2] are rock and fluid compressibility

suggests that $B = \frac{b}{(2\pi H)^2 r_w g}$

so field-scale estimates of b can be obtained from values of B.

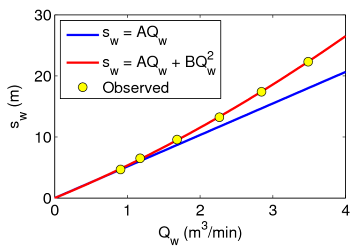


Figure 5: Plot of quasi-steady drawdown, s_w, against corresponding pumping rate, Q_w, for the step drawdown test data shown in Figure 6.

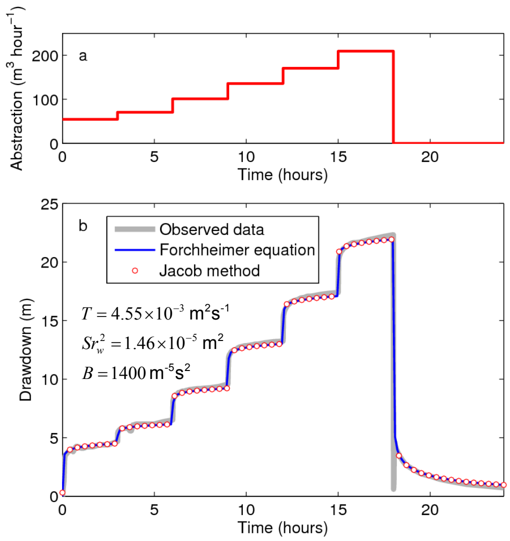


Figure 6: Step drawdown test in a confined sandstone aquifer after Clark (1977).

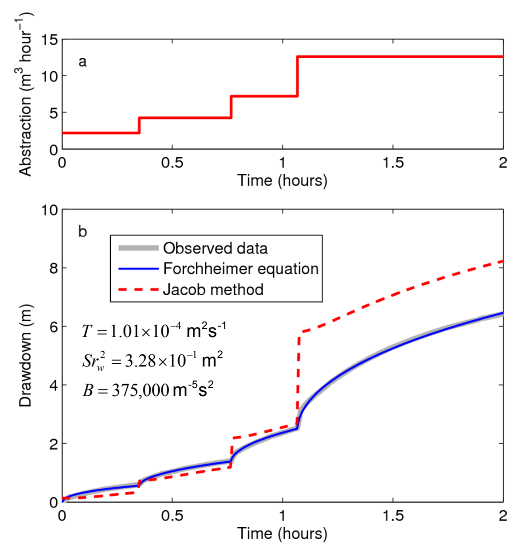


Figure 7: Step drawdown test in a fractured sandstone aquifer (after van Tonder, 2001).

Further reading

Mathias, SA & Todman, LC 2010. Step-drawdown tests and the Forchheimer equation. Water Resources Research 46: W07514.

Mathias, SA, Butler, AP & Zhan, HB 2008. Approximate solutions for Forchheimer flow to a well. Journal of Hydraulic Engineering - ASCE 134(9): 1318-1325.